JUSTIFY ALL ANSWERS!!!!   **COMMUNICATE EFFECTIVELY!!!**   **DUE DATE : First Day of School!!!!**

READ AND FOLLOW ALL DIRECTIONS CAREFULLY: This packet contains a review of most of the prerequisite skills required to be successful in AP Calculus. Use the space provided in this packet to **neatly** complete each problem. **Do not attach additional papers** - only this original packet will be scored, so if you do your work on other paper you must copy it carefully onto this packet.   All problems should be completed in pencil and, unless otherwise noted with a calculator icon, without the use of any calculator. When completing a question using the calculator, you are still expected to justify your work and provide any long decimal answers that must be rounded as the answer choice. If you do not understand a problem, or do not remember how to do a problem, you will need to use other resources to review/learn the material. **Justification** must be shown for each answer, as it is your responsibility to **effectively communicate** your solution to me. I am looking for your thought process that leads to your answer.   Correct answers without proper justification will not earn full credit. The letter choice for each problem must be written on the space provided before each question number. This packet is due on the first day of school, Monday, August 29th, 2022, when it will be collected and graded for accuracy.   It is my expectation that all solutions and answers will be neat and easy to understand. This packet will count as your first process grade of the quarter. After I have returned your scored packets, expect a QUIZ on these questions!

I am looking forward to a great year with each of you!!!!   This is where everything comes together!!!!

____ **1. Simplify:** $\left(19x^{-6}y^{11}\right)\left(-6xy^5\right)$

    a. $-114x^{-5}y^{16}$ b. $\dfrac{13y^{16}}{x^5}$ c. $\dfrac{-114y^6}{x^6}$ d. $-114x^{-7}y^{-24}$

____ **2. Simplify:** $\dfrac{4}{16+\sqrt{11}}$

    a. $\dfrac{64+4\sqrt{11}}{-245}$ b. $\dfrac{64+4\sqrt{11}}{245}$ c. $\dfrac{64-\sqrt{11}}{245}$ d. $\dfrac{64-4\sqrt{11}}{245}$

____ **3. Simplify:** $\left(y^{\frac{5}{12}}\right)\left(y^{\frac{1}{4}}\right)$

    a. $y^{\frac{15}{144}}$ b. $y^{\frac{2}{3}}$ c. $y^{\frac{1}{3}}$ d. $y^{\frac{5}{4}}$

____ **4. Simplify:** $\dfrac{3}{4x^2-25}+\dfrac{2}{2x+5}$

    a. $\dfrac{4x+7}{(2x+5)(2x-5)}$ b. $\dfrac{4x-10}{(2x-5)(2x+5)}$ c. $\dfrac{4x-7}{(2x+5)(2x-5)}$ d. $\dfrac{5}{(4x^2+2x-20)}$

____ **5. Simplify:** $\dfrac{8}{y+2}-\dfrac{3y}{y^2-4}$

    a. $\dfrac{5y-16}{(y+2)(y-2)}$ b. $\dfrac{8-3y}{(y+2)(y^2-4)}$ c. $\dfrac{5y-16}{(y+2)(y^2-4)}$ d. $\dfrac{5y+16}{(y+2)(y-2)}$

_____ 6. **Factor Completely:** $30x^3 - 50x^2 + 27x - 45$

     a. $(3x^2 - 9)(10x + 5)$ c. $10x^2(3x - 5)(27x + 45)$

     b. $(10x^2 + 9)(3x - 5)$ d. $(30x^2 - 1)(x - 45)$
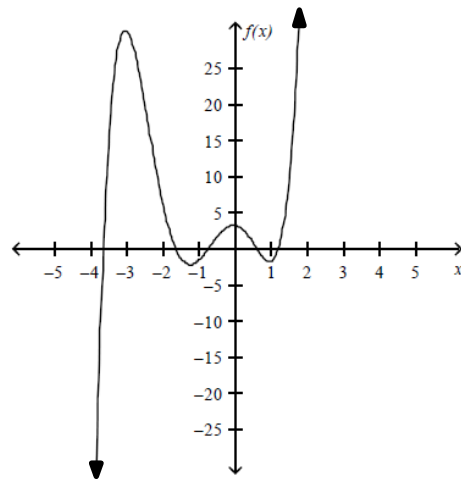

_____ 7. **Factor Completely:** $4x^2 - 13x + 9$

     a. $(2x - 9)(2x + 4)$ c. $(4x - 1)(x - 9)$

     b. $4x^2 - 3x - 10x + 9$ d. $(4x - 9)(x - 1)$

_____ 8. **Simplify:** $\dfrac{30x^2 + 53x + 22}{70x^2 + 17x - 66}$. **Assume that no denominator is equal to zero.**

     a. $\dfrac{3x-2}{7x+6}$ b. $\dfrac{3x-2}{7x-6}$ c. $\dfrac{3x+2}{7x-6}$ d. $\dfrac{3x+2}{7x+6}$

_____ 9. **Which of the following are true about the following graph? Justify your selection!**

     a. It is an odd-degree polynomial function.
        The function has five real zeros.
$$\lim_{x \to -\infty} f(x) = +\infty \quad \text{and} \quad \lim_{x \to +\infty} f(x) = +\infty$$

     b. It is an odd-degree polynomial function.
        The function has five real zeros.
$$\lim_{x \to -\infty} f(x) = -\infty \quad \text{and} \quad \lim_{x \to +\infty} f(x) = +\infty$$

     c. It is an odd-degree polynomial function.
        The function has four real zeros.
$$\lim_{x \to -\infty} f(x) = -\infty \quad \text{and} \quad \lim_{x \to +\infty} f(x) = +\infty$$

     d. It is an even-degree polynomial function.
        The function has five real zeros.
$$\lim_{x \to -\infty} f(x) = -\infty \quad \text{and} \quad \lim_{x \to +\infty} f(x) = +\infty$$

_____ 10. **For the given function, <u>state the real zeros</u> and <u>indicate where each one is located</u>.**
       $f(x) = -2x^4 - 4x^3 - 2x^2 + 3x + 8$ **(As always, give the long decimal value.)**

     a. There is only a zero between x = 1 and x = 2.

     b. There are zeros between x = 2 and x = 3, between x = 0 and x = 1, and between x = −2 and x = −3.

     c. There are zeros between x = 1 and x = 2 and between x = −1 and x = −2.

     d. There is only a zero between x = −1 and x = −2.

_____ **11. Estimate the x-coordinates at which the relative maxima and relative minima occur for the function**
$$f(x) = x^3 - 4x^2 - 11x + 30 \qquad \text{(As always, give the long decimal value.)}$$

a. The relative maximum is at x = −3 and the relative minimum is at x = 5 .

b. The relative maximum is at x = 36 and the relative minimum is at x = −14.815 .

c. The relative maximum is at x = 30 and the relative minimum is at x = −11 .

d. The relative maximum is at x = −1 and the relative minimum is at x = 3.667 .

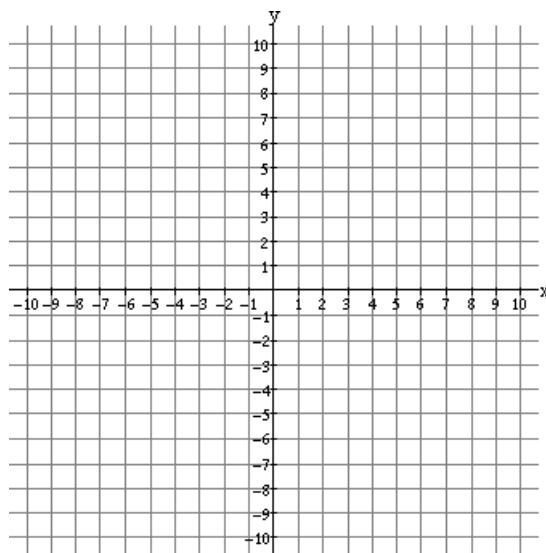_____ **12. Find all the zeros of the function $f(x) = x^3 - 15x^2 + 73x - 111$ (No calculator.)**

a. $3,\ 6+i,\ 6-i$

b. $-3,\ 3,\ 6+i$

c. $-3,\ 3,\ 6-i$

d. $-3,\ 6+i,\ 6-i$

_____ **13. Calculate the inverse of the function given by $f(x) = \dfrac{7x-3}{16}$**

a. $f^{-1}(x) = \dfrac{16x-3}{7}$

b. $f^{-1}(x) = \dfrac{16x+3}{7}$

c. $f^{-1}(x) = \dfrac{7x+16}{3}$

d. $f^{-1}(x) = \dfrac{7x-16}{3}$

_____ **14. Graph the function given by $f(x) = |2x + 6| - 5$,**
label three (x , y) coordinate points on f(x), including
the vertex, and determine the domain and range of f(x).

a. domain: $x \geq -3$
range: $y \leq -5$

b. domain: $\mathbb{R}$
range: $y \geq -5$

c. domain: $x \geq -6$
range: $y \leq -5$

d. domain: $\mathbb{R}$
range: $\mathbb{R}$

15. In ΔABC, m∠ABC = 90°, AC = 25 and BC = 20. Sketch and label ΔABC.
   Determine the value of sin (∠A) and cot (∠A).

   a. $\sin(\angle A) = \frac{3}{5}$ and $\cot(\angle A) = \frac{3}{4}$

   b. $\sin(\angle A) = \frac{4}{5}$ and $\cot(\angle A) = \frac{3}{4}$

   c. $\sin(\angle A) = \frac{4}{5}$ and $\cot(\angle A) = \frac{4}{3}$

   d. $\sin(\angle A) = \frac{3}{5}$ and $\cot(\angle A) = \frac{4}{3}$

_____ **16. In ΔJFK, m∠K = 114°, JK = 9 and FK = 16. Write an equation and calculate JF.**
   (As always, give the long decimal value.)

   a. 21.395

   b. 21.311

   c. 18.358

   d. 13.229

_____ **17. In ΔMAT, m∠M = 63°25'18", ∠A = 44°15'33"and TM = 20.612.**
   Write and equation and calculate AT.
   (As always, give the long decimal value.)

   a. 16.085

   b. 18.387

   c. 23.126

   d. 26.413

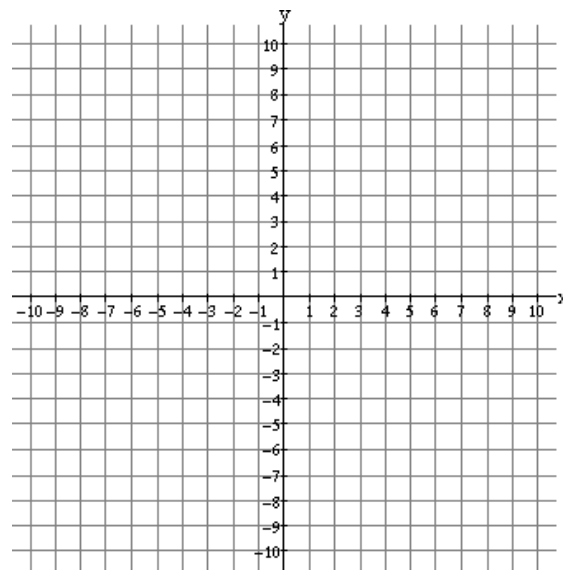_____ **18. Give the domain of the function given by $f(x) = \frac{x^2-9x+20}{x^2-4x+3}$.**

   a. $\mathbb{R}/x \neq 3, x \neq 1$

   b. $\mathbb{R}/x \neq -5, x \neq -4$

   c. $\mathbb{R}/x \neq -3, x \neq -1$

   d. $\mathbb{R}/x \neq -5, x \neq 4$

_____ 19. Calculate the domain of the function given by $f(x) = \sqrt{-2x - 4}$.

    a. $x \geq 0$

    b. $x \leq -2$

    c. $x \geq -2$

    d. $x \leq 0$

_____ 20. Graph the function given by $f(x) = -x^2 - 4$,
label three (x , y) coordinate points on $f(x)$
and determine the domain and range of $f(x)$.

    a. domain: $(-\infty, +\infty)$
range: $(-4, +\infty)$

    b. domain: $(-\infty, +\infty)$
range: $(-\infty, +\infty)$

    c. domain: $(-\infty, +\infty)$
range: $(-\infty, -4]$

    d. domain: $(-\infty, +\infty)$
range: $[-4, +\infty)$

_____ 21. Determine the vertical asymptotes(s), if any, for $f(x) = \frac{3x-7}{x^2-5x+6}$.

    a. $x = 7, x = 2$

    b. $x = 2, x = 3, x = 7$

    c. $x = 2, x = 3$

    d. No vertical asymptotes

_____ 22. Determine if the graph of the rational function has a slant asymptote.
If it does, find the equation of the slant asymptote.
$$f(x) = \frac{2x^3 + x^2 - 6x + 2}{x^2 - x - 2}$$

    a. $y = 2x$

    b. $y = 2x - 3$

    c. $y = 2x + 3$

    d. No slant asymptotes

_____ 23. Determine any horizontal asymptote of the graph given by $f(x) = \frac{2x^2+8}{3x^2+4x-1}$.

a. $y = \frac{2}{3}$ 　　　　b. $y = -8$ 　　　　c. $y = 0$ 　　　　d. No horizontal asymptote

_____ 24. Sketch the graph of the following rational function labeling three (x , y) coordinate points on $f(x)$ and any asymptotes.
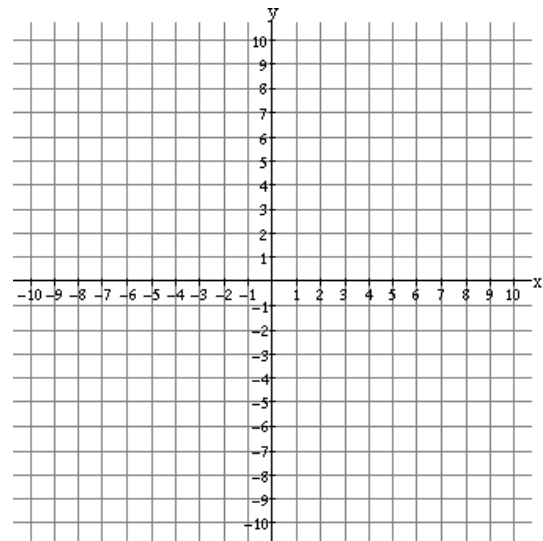
$$f(x) = \frac{2x^2-5x-3}{x^2-4}$$

a. asymptotes: $x = -3$, $x = 2$, $y = 3$

b. asymptotes: $x = -2$, $x = 2$, $y = 2$

c. asymptotes: $x = -3$, $x = 3$, $y = 2$

d. asymptotes: $x = -1$, $x = 3$, $y = 3$

_____ 25. Find $\frac{f(x+h)-f(x)}{(x+h)-(x)}$ for the function $f(x) = 3x^2 + 5$.

a. $3x^2 + 5 + h$

b. $6xh + 3h$

c. $3x + 3h$

d. $6x + 3h$

_____ 26. Evaluate the expression: $5\ln(e^2)$

a. $5 + e^2$ 　　　　b. $10e$ 　　　　c. 7 　　　　d. 10

_____ 27. Evaluate the expression: $2e^{ln(14)}$.

a. $\ln(28)$ 　　　　b. 28 　　　　c. 14 　　　　d. e^7

_____ **28. Solve the given equation:** $5e^{-0.04x} = 30$ **. (Justify and, as always, give the long decimal value.)**

a. -0.557

b. -44.794

c. -150.000

d. -19.454

_____ **29. Solve the given equation <u>algebraically</u>:** $\dfrac{600}{1+e^{-x}} = 500$ **.**

a. $\ln\left(\dfrac{6}{5}\right)$

b. $\ln(6)$

c. $\ln(5)$

d. $\ln\left(\dfrac{5}{6}\right)$

_____ **30. Find the value of x:** $3\ln(4x) = 13$ **. (Justify and, as always, give the long decimal value.)**

a. 19.049

b. 1.083

c. 17.333

d. 0.367

_____ **31. Condense and write** $3\ln(5) - \ln(2)$ **as a single logarithm?**

a. $\ln(7.5)$ b. $\ln(27)$ c. $\ln\left(\dfrac{5}{2}\right)^2$ d. $\ln(62.5)$

_____ **32. What is the solution of** $e^{x+1} = 13$ **?**

a. $x = \ln(13) + 1$

b. $x = \ln(13) - 1$

c. $x = \ln(13)$

d. $x = \ln(12)$

_____ 33. Sketch $\theta = \dfrac{4}{3}\pi$ in standard position. Label the reference triangle and determine $\sin\left(\dfrac{4}{3}\pi\right)$.

a. $-\dfrac{\sqrt{3}}{2}$

b. $\dfrac{2\sqrt{3}}{3}$

c. $\dfrac{1}{2}$

d. $-\dfrac{3\sqrt{3}}{3}$

_____ 34. Sketch $\theta = \dfrac{3}{4}\pi$ in standard position. Label the reference triangle and determine $\cos\left(\dfrac{3}{4}\pi\right)$.

a. -1

b. $-\dfrac{\sqrt{3}}{2}$
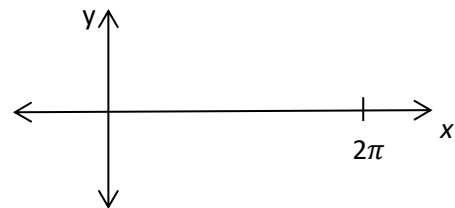
c. $\dfrac{\sqrt{2}}{2}$

d. $-\dfrac{\sqrt{2}}{2}$

_____ 35. Graph $y = \sin(\theta)$, $where\ 0 \le \theta \le 2\pi$.
   Use the graph to indicate $\sin\left(\dfrac{3\pi}{2}\right)$.

a. $-\dfrac{\sqrt{2}}{2}$ b. -1 c. 0 d. 1

_____ 36. Determine the value of θ if $\sin(\theta) = 1$, $where\ 0 \le \theta \le 2\pi$. Justify by labeling the above graph.
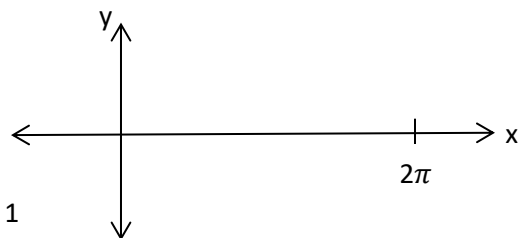
a. $\dfrac{\pi}{2}$ b. $\dfrac{\pi}{3}$ c. $\dfrac{\pi}{4}$ d. Not possible

_____ 37. Graph $y = \cos(\theta)$, *where* $0 \le \theta \le 2\pi$.
Use the graph to indicate $\cos\left(\frac{3\pi}{2}\right)$.

a. $-\frac{\sqrt{2}}{2}$ b. -1 c. 0 d. 1

_____ 38. Determine the value of θ if $\cos(\theta) = -1$ *where* $0 \le \theta \le 2\pi$. Justify by labeling the above graph.

a. $\frac{\pi}{2}$ b. $\frac{\pi}{4}$ c. 0 d. π

_____ 39. Evaluate $\tan\left(Cos^{-1}\left(\frac{1}{2}\right)\right)$. Provide a sketch of a triangle to justify your answer.

a. $\frac{\sqrt{3}}{3}$ b. $\sqrt{3}$ c. $\frac{\sqrt{3}}{2}$ d. $\frac{\sqrt{2}}{2}$

_____ 40. Calculate the area of a triangle whose sides measure 8cm, 14cm and 20cm.
(Justify and, as always, give the long decimal value.)

a. $43.715 \; cm^2$

b. $47.329 \; cm^2$

c. $56 \; cm^2$

d. $140 \; cm^2$

_____ 41. Calculate the x-values that satisfy the equation: $8\cos(x) - 4\sqrt{3} = 0$, *where* $0 \le x \le 2\pi$.

a. $\frac{\pi}{6}, \frac{11\pi}{6}$

b. $\frac{5\pi}{6}, \frac{7\pi}{6}$

c. $\frac{7\pi}{6}, \frac{11\pi}{6}$

d. $\frac{\pi}{6}, \frac{7\pi}{6}$

_____ 42. **John invested $5000 at 2.8 % apr compounded continuously.**
Write an equation and determine, to the nearest dollar, what John's investment was
worth after 3 years?   (Justify and, as always, give the long decimal value.)

a. $ 5084

b. $ 5280

c. $ 5438

d. $ 5840

_____ 43. **How many years (nearest thousandths) does it take to double an investment compounded**
continuously at 4.5 % apr? (Justify and, as always, give the long decimal value.)

a. 7.702 years

b. 15.403 years

c. 22.222 years

d. 36.389 years

_____ 44. **Divide** $\left(8x^4 - 20x^3 - 14x^2 + 8x + 1\right)$ $\boldsymbol{by}$ $(x + 1)$ **using synthetic division.**
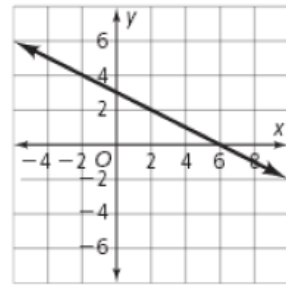
a. $8x^3 + 28x^2 + 14x - \dfrac{17}{x+1}$

b. $8x^3 + 36x^2 + 18x + 10 + \dfrac{11}{x+1}$

c. $8x^3 - 28x^2 + 14x - 6 + \dfrac{7}{x+1}$

d. $8x^3 + 28x^2 + 14x + 8$

_____ 45. Which equation represents a line through $(-1, 1)$ and $(2, 3)$?

  a. $y = \dfrac{3}{2}\,x$

  b. $y = 2\,x - 1$

  c. $y = \dfrac{2}{3}\,x + \dfrac{5}{3}$

  d. $y = -\dfrac{2}{3}\,x + \dfrac{13}{3}$

_____ 46. Which of the following equations is shown in the following graph?

  a. $y + 2 = -\dfrac{1}{2}\,(x + 2)$

  b. $y + 3 = -\dfrac{1}{2}\,(x + 6)$

  c. $y - 3 = -\dfrac{1}{2}\,(x - 6)$

  d. $y - 2 = -\dfrac{1}{2}\,(x - 2)$

_____ 47. Which of the following systems of equations has the solution $(4, -1)$?

  a. $-2x + 4y = 6 \ and -3x + 6y = -2$

  b. $3x - y = 0 \ and\ 4x + 3y = 26$

  c. $3x - 2y = 14 \ and\ 2x + 2y = 6$

  d. $4x + 9y = 1 \ and\ 4x + 6y = -2$

_____ 48. Give the meaning of $\displaystyle\sum_{i=5}^{8} (3i + 4)$

  a. $7, 10, 13, 16, 19, 22, 25, 28$

  b. $19 + 22 + 25 + 28 = 94$

  c. $(3(5) + 4) + (3(8) + 4) = 47$

  d. $3\dfrac{(8)(8+1)}{2} = 108$

_____ 49. Determine the number of terms in the finite arithmetic sequence: -3, 1, 5, 9, 13, … 825.

    a. 208

    b. 275

    c. 825

    d. 828

_____ 50. **A certain type of radioactive element has a half-life of 15 years.**
Suppose that the initially there were 22 grams. Write an equation and determine the number
of years it would take for only 8.5 grams to remain.
(Justify and, as always, give the long decimal value.)

    a. 1.467 years

    b. 13.864 years

    c. 18.942 years

    d. 20.580 years

DID YOU JUSTIFY ALL OF YOUR ANSWERS???????

☆ BONUS ☆

Given $f(x) = \frac{x-2}{x+1}$. determine $f'(x)$ then evaluate $f'(-2)$ and explain the significance of $f'(-2)$.

☆ BONUS ☆

Given $f(x) = x^2 - 4x + 7$, determine the equation of the tangent line @ $x = 2$.